TURBULENT-SKIN-FRICTION AND HEAT-TRANSFER CHARTS ADAPTED FROM THE SPALDING AND CHI METHOD

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SUMMARY

Charts are presented which allow a rapid determination of local and average skin friction and heat transfer on flat plates in air. The charts cover a Mach number range from 0 to 20, a Reynolds number range from 10^5 to 10^9 in decade increments of the exponent, and a wall-temperature—stagnation-temperature ratio range of 0.1 to the adiabatic-wall case.

INTRODUCTION

In the design of large hypersonic vehicles, the need for accurate estimates of turbulent skin friction and heat transfer is important. A number of prediction methods have accumulated over the years, but the results from these methods often differ widely and recourse to experiments must be made. In early 1964, Spalding and Chi (ref. 1) compared experimental skin friction with 20 different turbulent theories. A theory of Van Driest (ref. 2), employing the Karman mixing length, was found to yield the lowest root-mean-square error (11.0 percent). However, Spalding and Chi were able to formulate a method applicable to a flat plate that gave an error of 9.9 percent. In a paper by Bertram and Neal (ref. 3), a body of heat-transfer data in the Mach number range from about 5 to 9 and wall-temperature—stagnation-temperature ratios down to about 0.1 was found to support the predicted effect of wall-temperature—stagnation-temperature ratio from the Spalding-Chi method. Because of current interest in the Spalding-Chi method, it was decided to present, in convenient graphical form, the results of extensive calculations that were accumulated while preparing reference 3.

SYMBOLS

C_F average skin-friction coefficient based on conditions at edge of boundary layer

C_f local skin-friction coefficient based on conditions at edge of boundary layer

$F_c, F_{Rx}, F_{R\theta}$ functions in the Spalding-Chi theory given by equations (3), (2), and (1), respectively

M Mach number at edge of boundary layer

N_{Pr} Prandtl number

N_{St} local Stanton number based on conditions at edge of boundary layer

R_x Reynolds number based on distance x and conditions at edge of boundary layer

T absolute temperature

$t = T/T_e$

Subscripts:

aw   adiabatic wall

i   incompressible value

w   wall

t   total

e   edge of boundary layer

DISCUSSION

In developing their method, Spalding and Chi postulated that a unique relation exists between $F_c C_f$ and $F_R R$ where C_f is the local skin-friction coefficient, R is the Reynolds number, and F_c and F_R depend only on Mach number and wall-temperature—stream-temperature ratio. The function F_c was calculated by means of the mixing-length theory, and the function F_R was deduced semi-empirically. Thus, values of skin friction could be obtained but heat transfer was not considered.

Tables and charts for F_c and F_R to be used in the calculation of skin friction were presented; however, for extensive use of the theory, it was found more convenient to

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compute values of F_c and F_R. When the Reynolds number is based on momentum thickness, $F_R = F_{R\theta}$ and is given by the empirical expression (eq. (52) of ref. 1).

$$F_{R\theta} = t_w^{-1.474} t_{aw}^{0.772} \tag{1}$$

or when based on length of turbulent flow x, $F_R = F_{Rx}$ and is given by equation (25) of reference 1.

$$F_{Rx} = \frac{F_{R\theta}}{F_c} \tag{2}$$

Only the Reynolds number based on distance x is considered in this paper. To compute F_c, equation (39) of reference 1 was integrated to yield the expression

$$F_c = \left(t_{aw} - 1\right)\left\{\sin^{-1} \frac{2\sqrt{t_{aw} - 1}\left[t_{aw} - t_w - \sqrt{t_w}(2 - t_{aw} - t_w)\right]}{\left(t_{aw} + t_w\right)^2 - 4t_w}\right\}^{-2} \tag{3}$$

The proper quadrant in which to evaluate the inverse sine is shown in the following sketch:
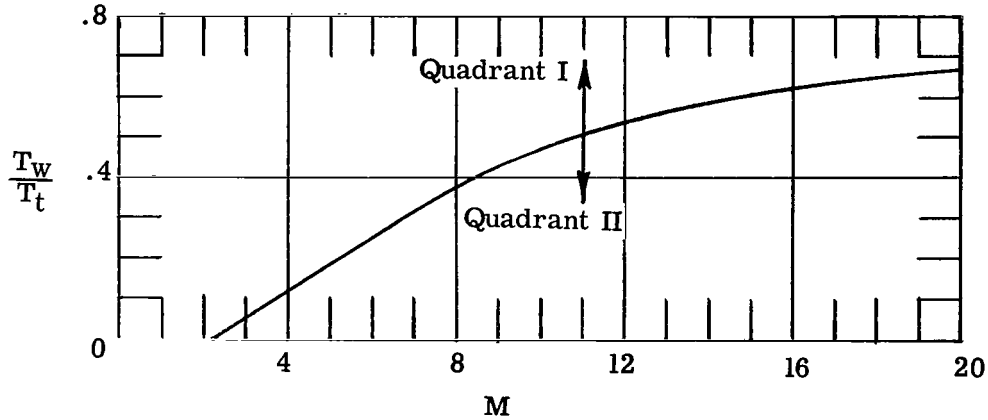
In the limit as M approaches 0, equation (3) reduces to

$$F_c = \left(\frac{1 + \sqrt{t_w}}{2}\right)^2 \tag{4}$$

By using these expressions and the variation of $F_c C_f$ and $F_c C_F$ with $F_{Rx} R_x$, as given in table 7 of reference 1, and extended where necessary by equations (28) to (30)

of reference 1*, values of C_f and C_F were computed. Stanton numbers were calculated by the modified Spalding-Chi method described in appendix A of reference 3. This approach consists of applying the Karman form of the Reynolds analogy in the incompressible plane so that the Karman equation is modified to be

$$\frac{2N_{St}}{C_f} = \left[1 + 5\sqrt{\frac{F_c C_f}{2}}\left(N_{Pr} - 1 + \log_e \frac{5N_{Pr} + 1}{6}\right)\right]^{-1} \tag{5}$$

When the Karman factor was applied in this paper, N_{Pr} was taken as 0.725.

Although directly applicable to a flat plate, the results for local skin friction and heat transfer may be applied to a cone with a known or assumed location of transition by using the transformation given in appendix B of reference 3. Results given in reference 4 (pp. 25 to 30) may aid in the application of the results to obtain the average skin friction on a cone or tapered wing.

PRESENTATION OF RESULTS

Charts of $C_f/C_{f,i}$, $N_{St}/N_{St,i}$, and $C_F/C_{F,i}$ are presented in figure 1 as a function of Mach number for $R_x = 10^5$ to 10^9 and $T_w/T_t = 0.1$ to the adiabatic-wall value. The recovery factor was taken to be 0.89. The charts are normalized by using the incompressible values of skin-friction coefficient given by the Karman-Schoenherr formulas:

$$\left.\begin{aligned}
\frac{0.242}{\sqrt{C_{F,i}}} &= \log_{10}\left(C_{F,i}R_x\right) = \log_{10}\left(2R_\theta\right)\\[2mm]
C_{f,i} &= \frac{0.242 C_{F,i}}{0.242 + 0.8686\sqrt{C_{F,i}}}
\end{aligned}\right\} \tag{6}$$

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*Equation (29) of reference 1 actually contains two typographical errors; the correct form which was determined by referring back to the paper in which the expression was originally presented is

$$R_{x,i} = \frac{1}{12}\left(u_G^+\right)^4 + \left(K^3 E\right)^{-1}\left\{\left[6 - 4\left(K u_G^+\right) + \left(K u_G^+\right)^2\right]\exp\left(K u_G^+\right) - 6 - 2\left(K u_G^+\right)\right.$$

$$\left. - \frac{1}{12}\left(K u_G^+\right)^4 - \frac{1}{20}\left(K u_G^+\right)^5 - \frac{1}{60}\left(K u_G^+\right)^6 - \frac{1}{252}\left(K u_G^+\right)^7\right\}$$

where $u_G^+ = \left(\frac{2}{C_f}\right)^{1/2}$, $K = 0.4$, and $E = 12$.

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The normalizing values of $N_{St,i}$ are those obtained from the unmodified Karman equation (eq. (A4) of ref. 3) in conjunction with equation (6) and are presented in figure 2.

In figure 1 for the adiabatic-wall cases at $M = 0$, the value of unity is not necessarily obtained because the Karman-Schoenherr formulas were used for normalizing rather than the incompressible values of Spalding and Chi. Actually, the incompressible values of skin-friction coefficients of Spalding and Chi do not differ significantly from those of Karman-Schoenherr as seen in figures 3 and 4, but because of the widespread use of the Karman-Schoenherr formulas in normalizing experimental data in past publications, they were chosen for use here. For some cases, the skin-friction or heat-transfer ratio drops below the value 0.1 before Mach number 20 is attained. Extrapolation is aided by noting that the logarithm of these ratios is essentially linear with the logarithm of M between Mach numbers of 10 and 20 at the high Reynolds numbers. Thus, extrapolation may be simply done on log-log paper.

It should be noted that at the higher Mach numbers and lowest Reynolds numbers, the equivalent incompressible Reynolds numbers are much lower than those expected for turbulent flow to exist. (See ref. 5.)

CONCLUDING REMARKS

Utilizing the Spalding-Chi turbulent-skin-friction theory, charts are presented which allow a rapid determination of the local and average skin friction on flat plates in air. An extension was made to allow the determination of heat transfer by modifying Spalding-Chi's skin-friction results according to the Karman form of the Reynolds analogy factor. The charts cover a Mach number range from 0 to 20, a Reynolds number range from 10^5 to 10^9 in decade increments of the exponent, and a wall-temperature—stagnation-temperature ratio range of 0.1 to the adiabatic-wall case.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., January 13, 1967,
129-01-09-14-23.

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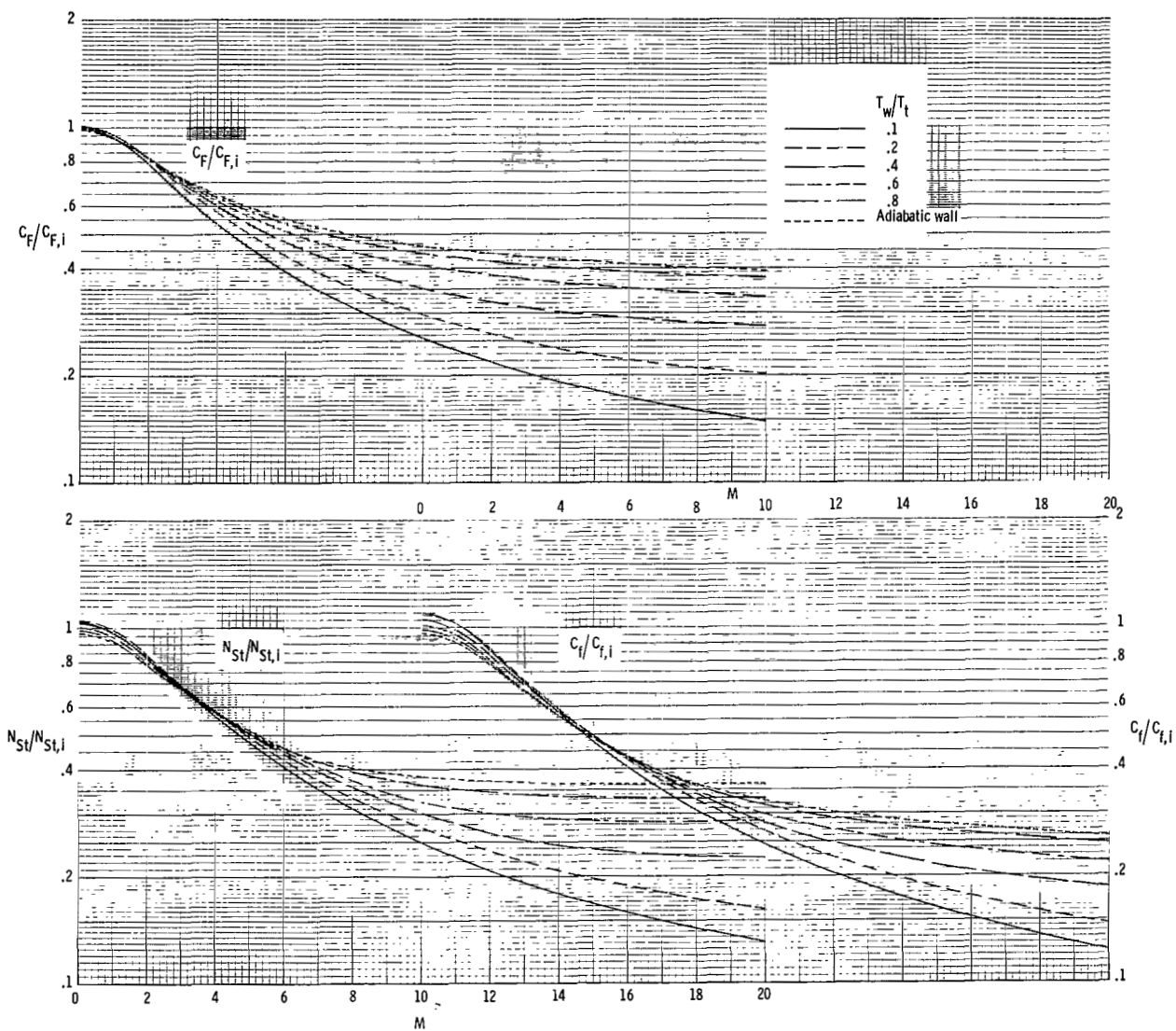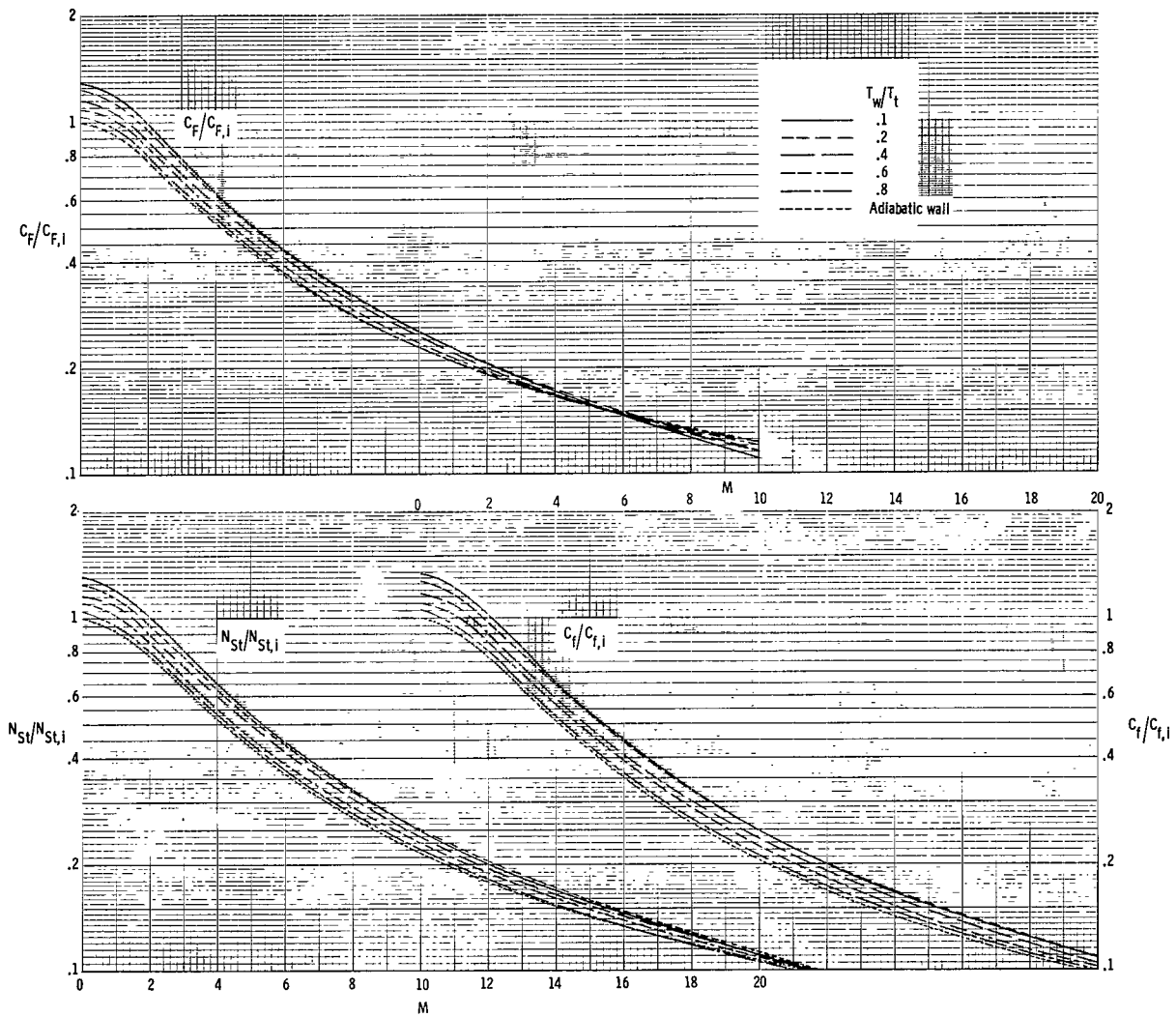
(a) $R_X = 10^5$.

Figure 1.- Variation of $C_f/C_{f,i}$, $N_{St}/N_{St,i}$, and $C_F/C_{F,i}$ as a function of Mach number for various values of wall-temperature—stagnation-temperature ratio.
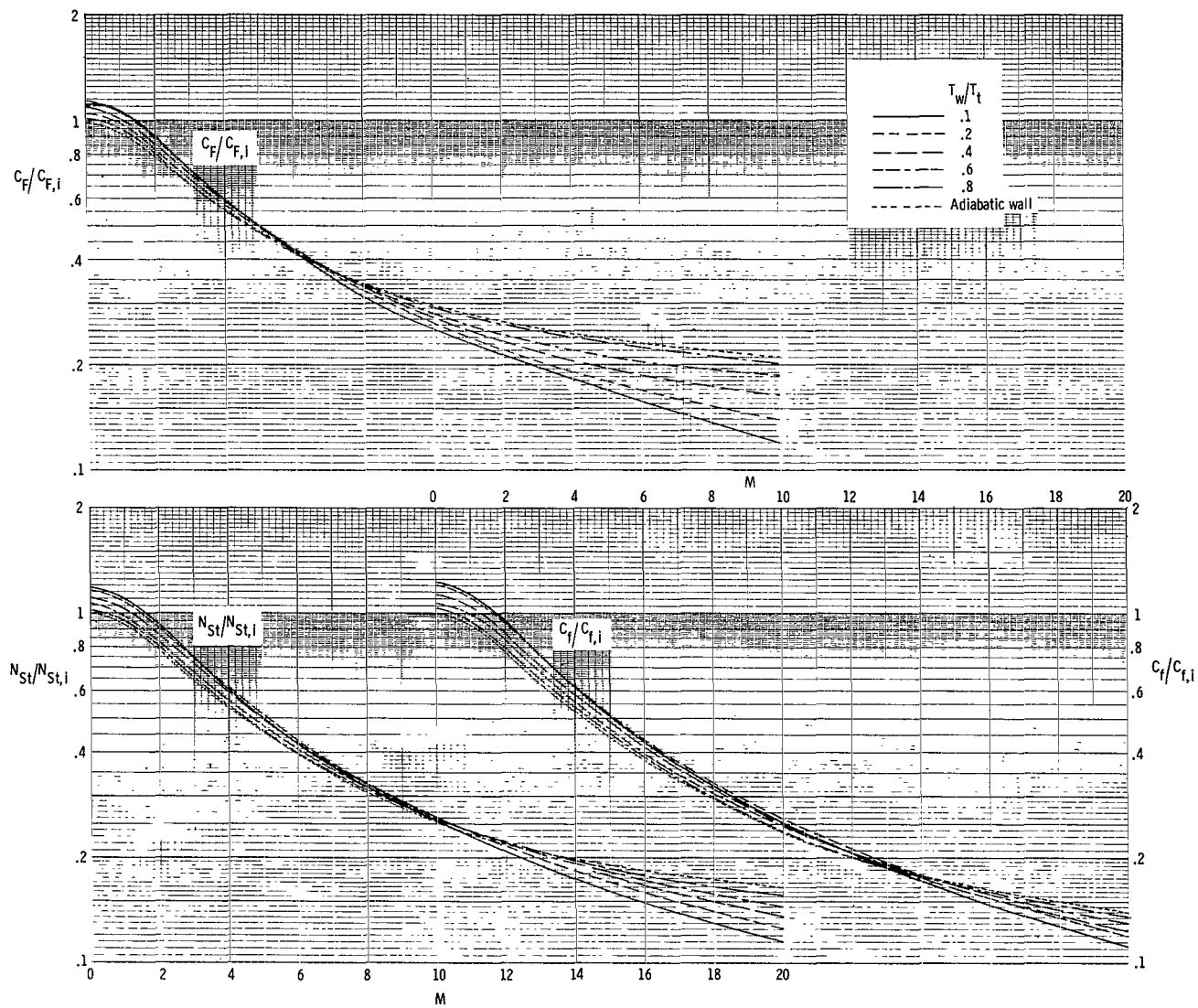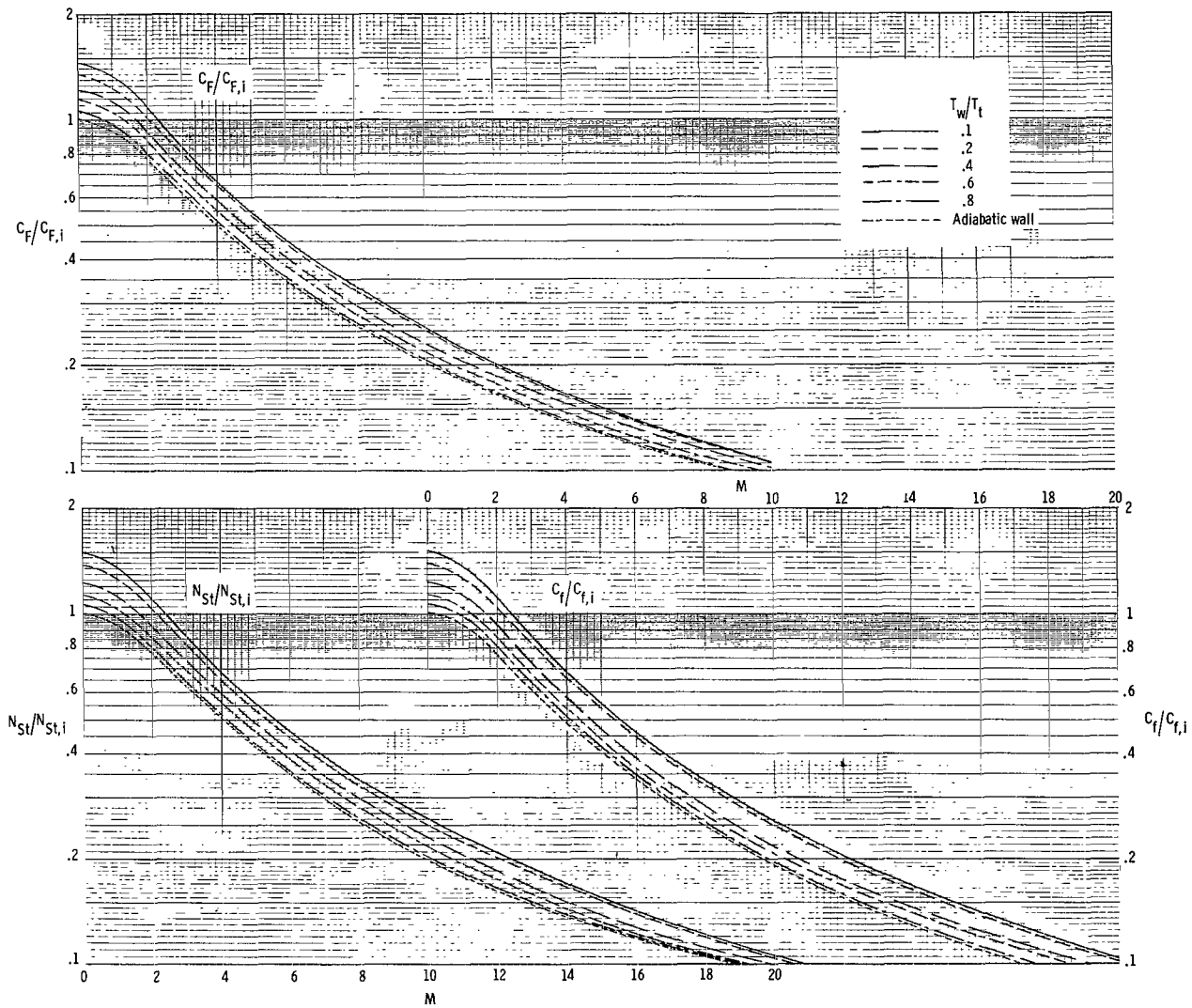
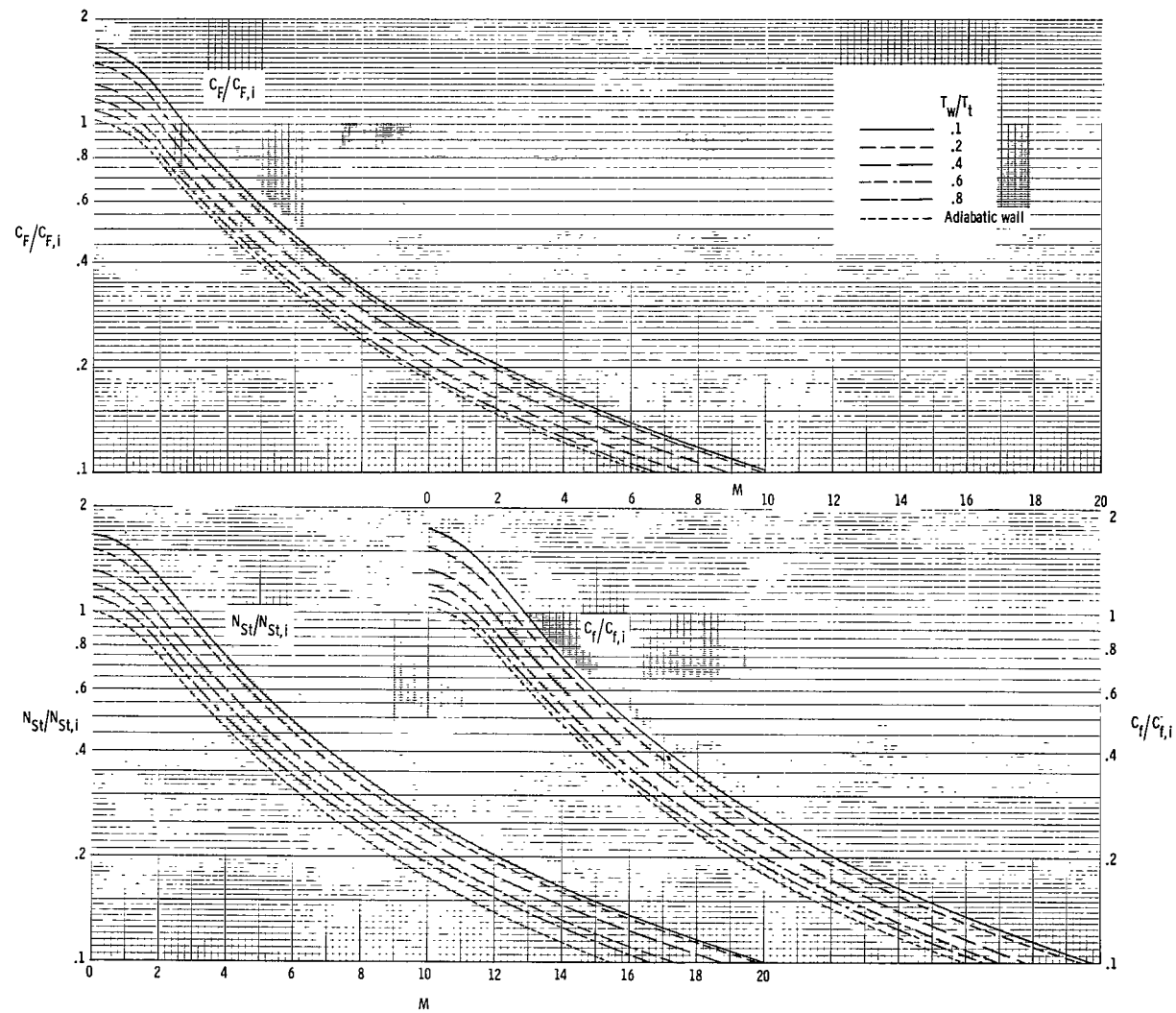(b) $R_X = 10^6$.

Figure 1.- Continued.

(c) $R_x = 10^7$.

Figure 1.- Continued.

(d) $R_x = 10^8$.

Figure 1.- Continued.

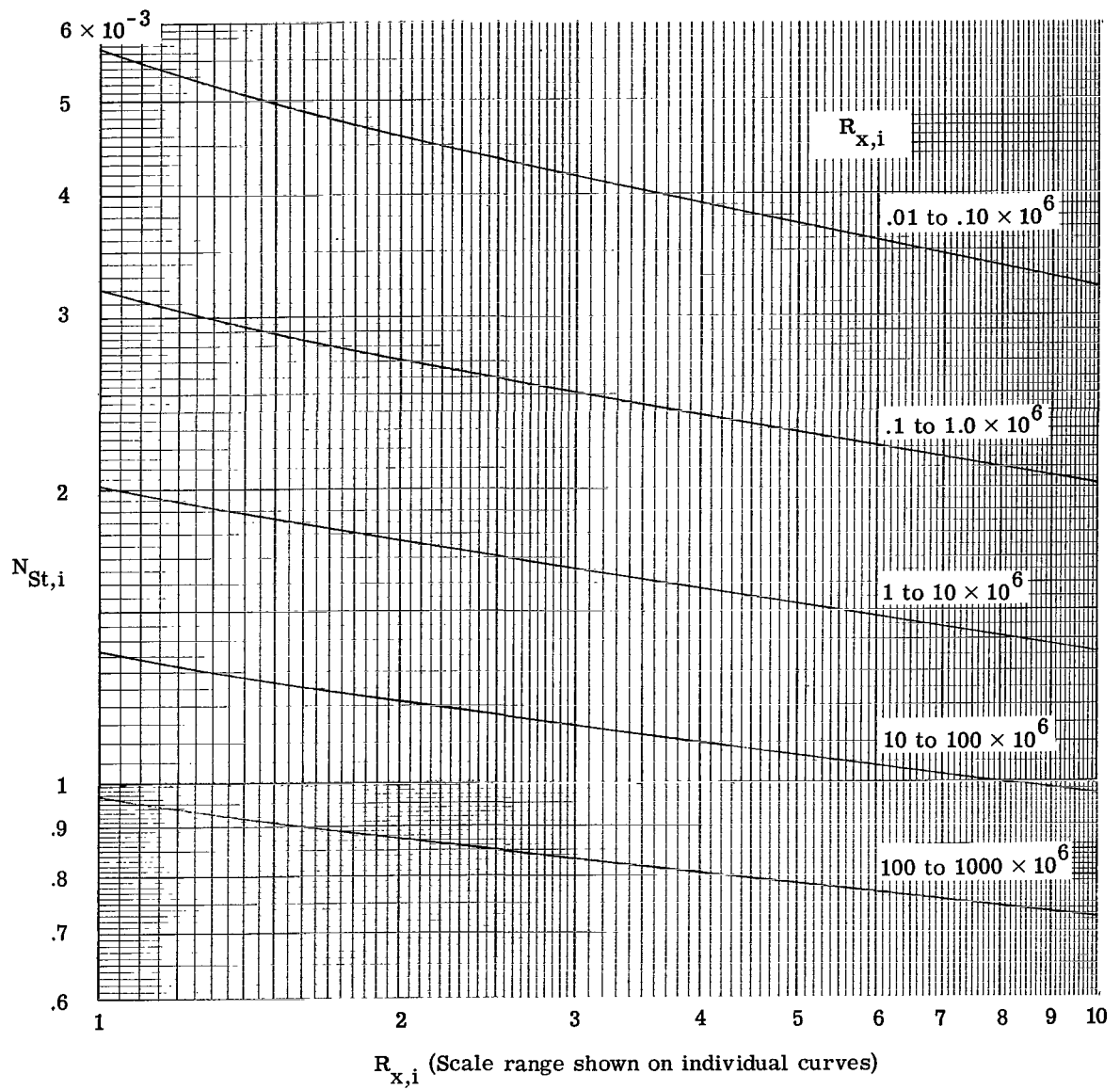(e) $R_x = 10^9$.

Figure 1.- Concluded.

Figure 2.- Variation of incompressible Stanton number with Reynolds number based on the Karman form of the Reynolds analogy equation and the Karman-Schoenherr skin-friction equation. $N_{Pr} = 0.725$.
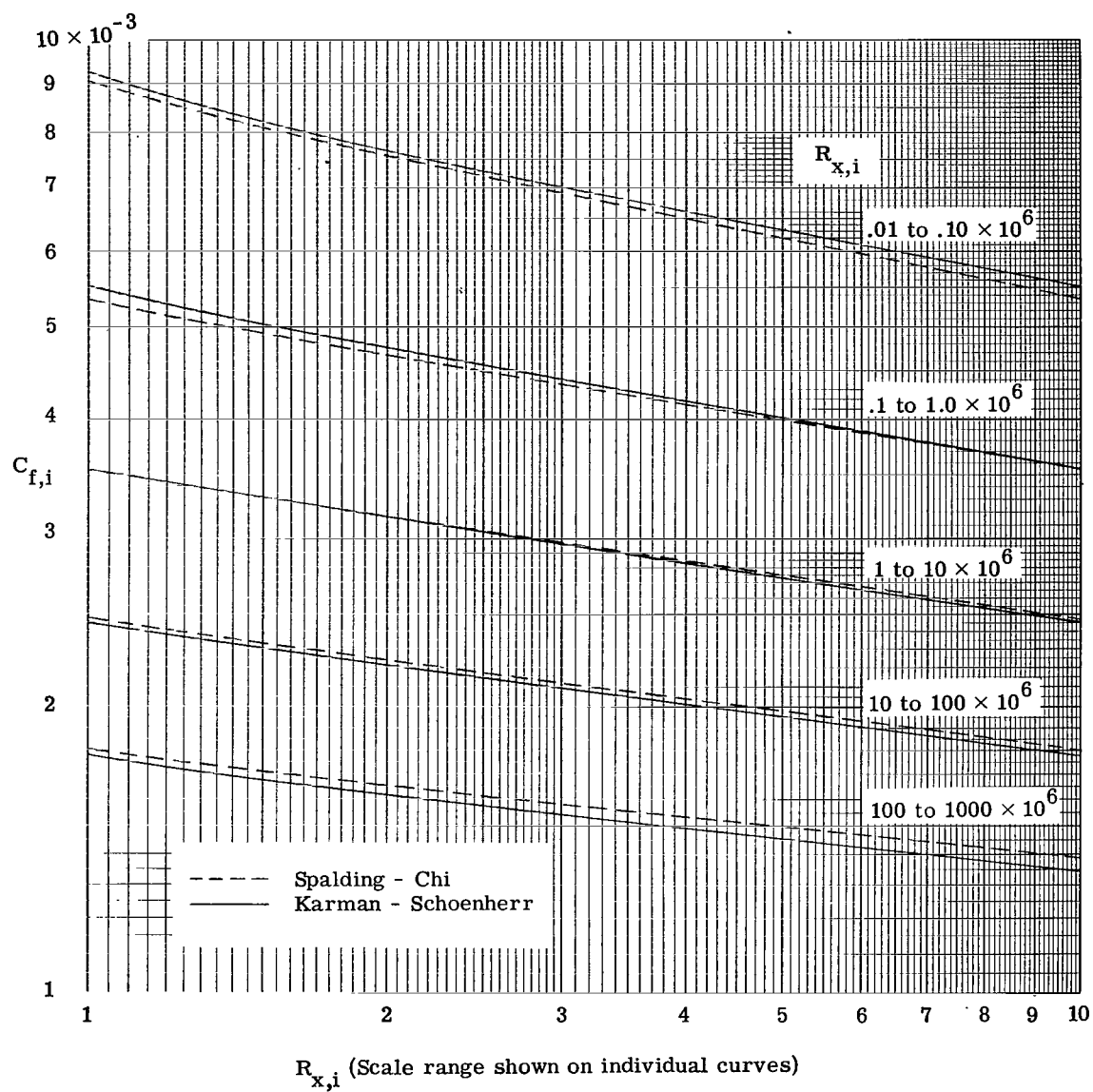
Figure 3.- Variation of incompressible local skin-friction coefficient with Reynolds number.
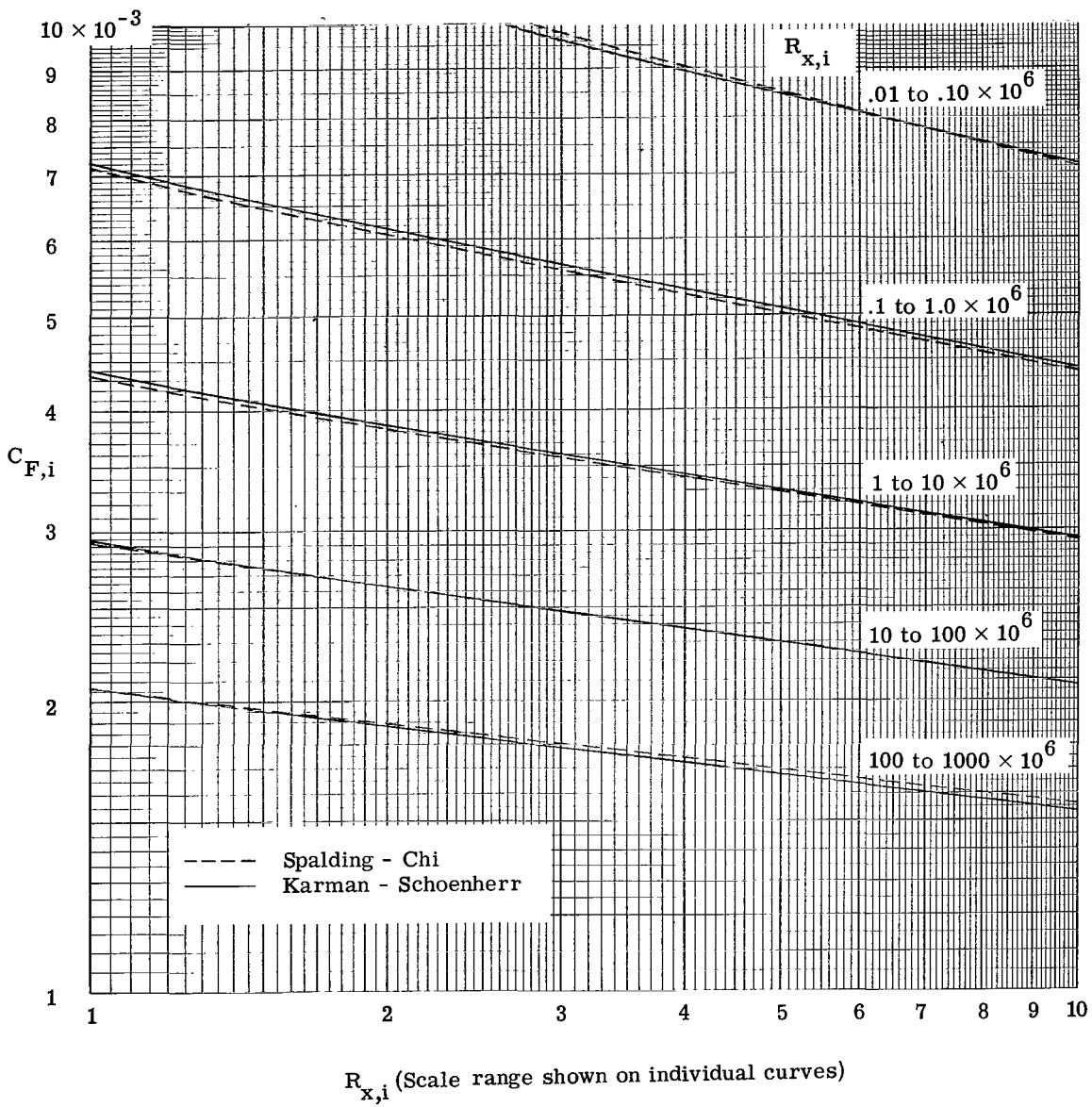
Figure 4.- Variation of incompressible average skin-friction coefficient with Reynolds numbers.

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